RAILWAY BRIDGES DAMPING IDENTIFICATION USING TRAFFIC INDUCED VIBRATION

Plan

ACOUSTIQUE VIBRATIONS LOGICIEL SCIENTIFIQUE

- Introduction
- Description of parametric models
- Models validation
- Application to real life signals
- Damping identification along decay
- Conclusion

Introduction

- Work commissioned and supported by ERRI committee D 214 'Railways bridges for speeds >200 km/h'
- Common methods of damping measurement are :
  - logarithmic decrement or general free decay analysis
  - frequency response under controlled excitation (sinus or transient)
- Because of their low cost, free decay analysis methods are prefered
  - due to the non stationarity of train excitation, signal processing problems occur : spectral methods are inefficient
  - the time methods are well adapted to solve these problems

Introduction

- Time methods are called parametric models
  - commonly used in speech processing and automatics since the late 60 's
  - the time signal is used to identify model parameters
  - the parametric models proposed here are
    - AR model (Auto Regressive)
    - Eigen value model (Prony Pisarenko)
  - the logarithmic decrement method was also tested

AR model

- sampled general linear system of input e_t and ouput x_t may be modeled by difference equation

$$x_t + a_1 x_{t-1} + \quad \dots \quad + a_{2m} x_{t-2m} = e_t \qquad\qquad x_t = -\sum_{k=1}^{2m} a_k x_{t-k} + e_t$$

or in matrix form

$$\begin{bmatrix} x_{2m+1} \\ x_{2m+2} \\ \vdots \\ x_t \end{bmatrix} = \begin{bmatrix} x_{2m} & x_{2m-1} & \cdots & x_1 \\ x_{2m+1} & x_{2m} & \cdots & x_2 \\ \vdots & \vdots & & \vdots \\ x_{t-1} & x_{t-2} & \cdots & x_{t-2m} \end{bmatrix} \begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_{2m} \end{bmatrix} + \begin{bmatrix} e_{2m+1} \\ e_{2m} \\ \vdots \\ e_t \end{bmatrix}$$

Parametric models

- a_k are called the AR coefficients
- These coefficients are related to the frequencies ω_k and dampings ζ_k by the relationships (Δt=sampling interval)

$$\omega_k \zeta_k = -\frac{Ln\left(\left|a_k\right|^2\right)}{2\Delta t}$$

$$\omega_k \sqrt{1-\zeta_k^2} = \frac{a\tan\left(\Im(a_k)/\Re(a_k)\right)}{\Delta t}$$

Parametric models

- The solution is found through least square estimation

$$X = \Phi_x \theta + E \qquad\qquad \theta = \left(\Phi_x^T \Phi_x\right)^{-1} \Phi_x^T X$$

- One cannot fully justify the use of the AR model unless the input of the system consists in white noise, which is not the case here

- The least square estimation is a source of bias errors in parameter estimation

The eigen value method (Prony-Pisarenko)

- When no input is present system equation can be written as $$x_t + \sum_{k=1}^{2m} a_k x_{t-k} = 0$$

$$\left[X \;\; \Phi_x\right] \begin{bmatrix} 1 \\ -\theta \end{bmatrix} = \tilde{\Phi}_x \begin{bmatrix} 1 \\ -\theta \end{bmatrix} = \{0\}$$

$$\tilde{\Phi}_x^T \tilde{\Phi}_x \begin{bmatrix} 1 \\ -\theta \end{bmatrix} = R_{xx} \begin{bmatrix} 1 \\ -\theta \end{bmatrix} = \{0\}$$

Parametric models

- R_{xx} is the covariance matrix
  - parameter vector $[1\ -\theta]^T$ is the covariance matrix eigen vector associated with eigen value 0
  - as R_{xx} is definite semi positive, this eigen value is the lowest one
  - it can be shown that in presence of noise at the output, the eigen value remains the lowest one

- The method simply consists in computing the covariance matrix R_{xx} and then calculating eigen vectors. The eigen vector whose eigen value is the lowest gives the a_k parameters

Models validation

- Simulations have been performed to validate the different models and estimate errors in damping estimation
  - AR, eigen model and logarithmic decrement have been tested
  - all methods are sensitive to
    - size (number of samples) of data processed
    - sampling frequency
    - filtering
    - modal coupling
  - The simulation aim is to optimize the choice of signal processing parameters

Models validation

- logarithmic decrement
  - bias errors are small < 5%
  - more than 10 periods of signal are necessary to obtain less than 15 % random error
  - in the case of several modes, the filtering around each mode gives similar results with less than 20 % random error
  - results are coherent when the modes are sufficiently separated
- AR & Eigen models
  - ~10 periods of signal are sufficient
  - AR model is biased but the filtering reduces this effect
  - eigen model is unbiased (<5%)
  - in these conditions bias and random errors are less than 10 %
  - with three modes bias error may increase (15 %) when coupling between modes is important (4% damping)

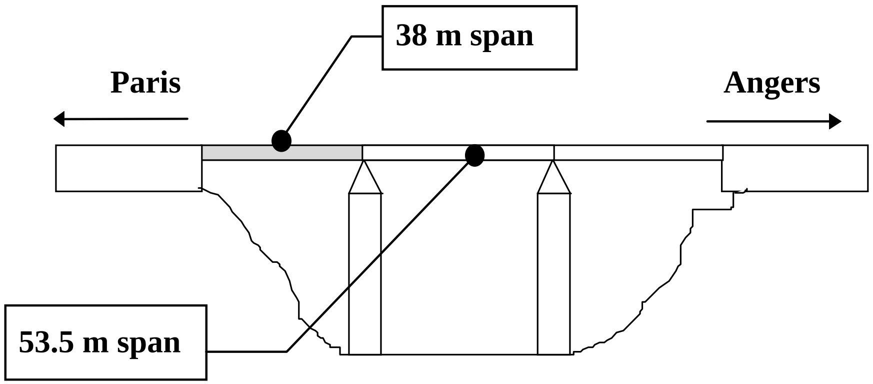
Models validation

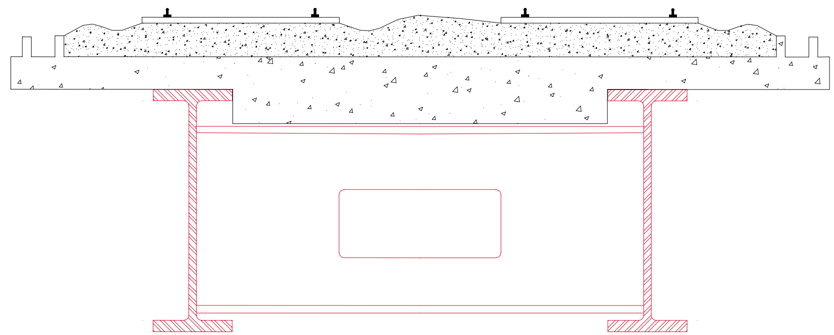
Conclusion

- logarithmic decrement is a good estimator, if the modes are not too close one to the other
- AR & Eigen models give good results, The Eigen model gives better results than AR model since it is unbiased
- simulations gave confidence in time methods. A strategy of mode identification is well defined and may be applied to real life measurements
- as errors in simulations are less than 15 %, greater errors must be expected in reality

Application to real life signals

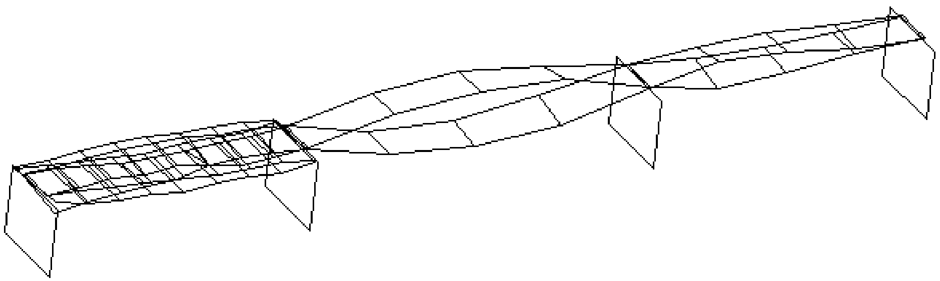
- Train passing by were recorded on Briollay bridge on TGV Atlantique high speed line
  - 3 spans steel/concrete bridge



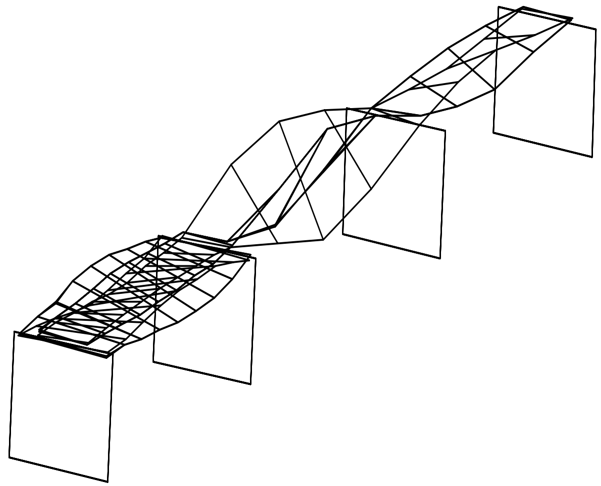


Application to real life signals

- Modal analysis was performed with hammer excitation

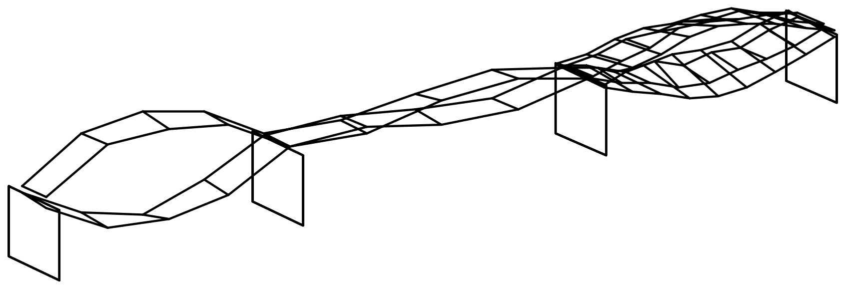


whole bridge flexion mode @ 2.29 Hz

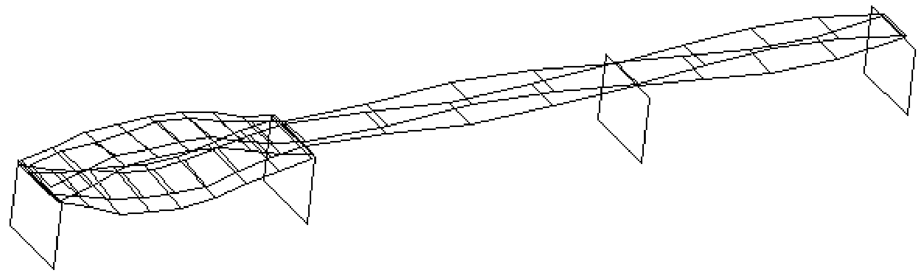


whole bridge torsion mode @ 3.28 Hz

Application to real life signals

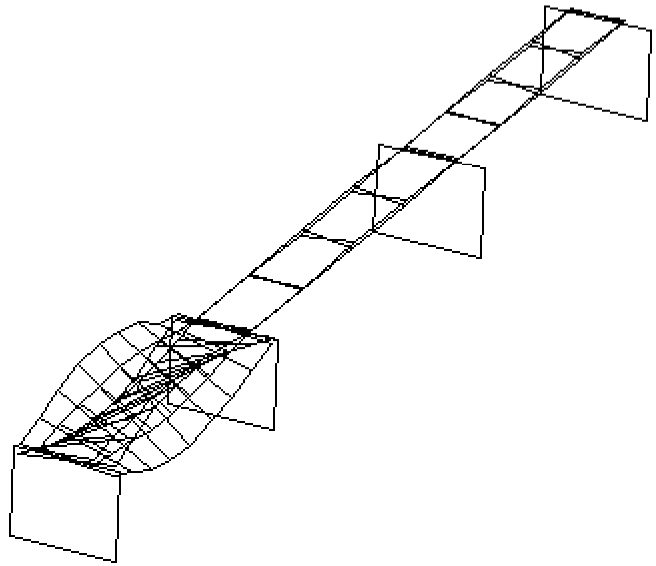


whole bridge flexion mode @ 3.66 Hz



1st span flexion mode @ 4.16 Hz

Application to real life signals

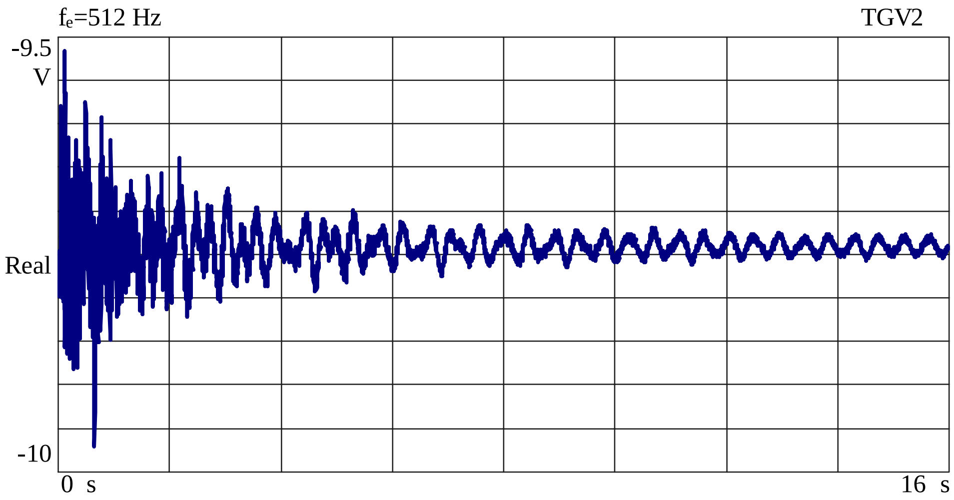


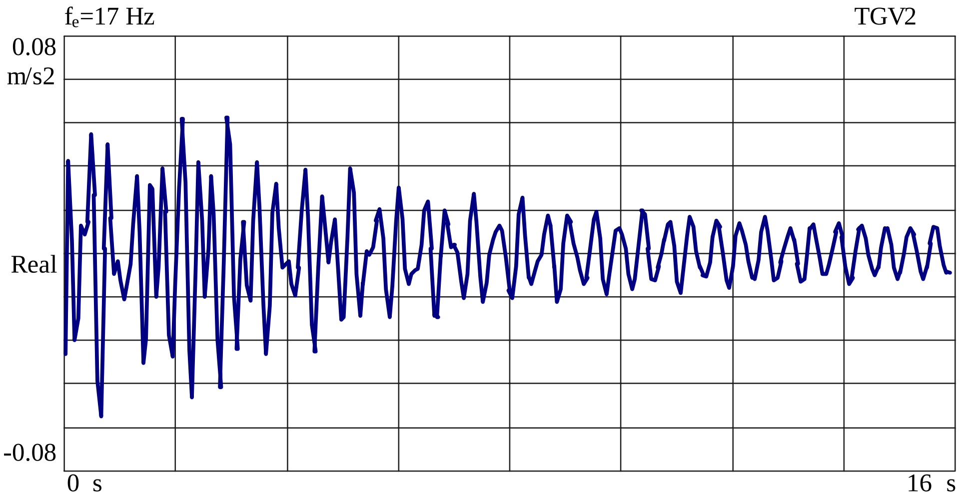
1st span torsion mode @ 4.96 Hz

Application to real life signals

- 6 train passing by are analyzed
  - 5 TGV
  - 1 TER
- Strategy for analyzing decay signals
  - measure the spectral components of the signal
  - separate modes to be identified in small groups
  - for each group, apply optimum sampling and filtering to isolate modes
  - identify frequency and damping

Application to real life signals





measurement signal

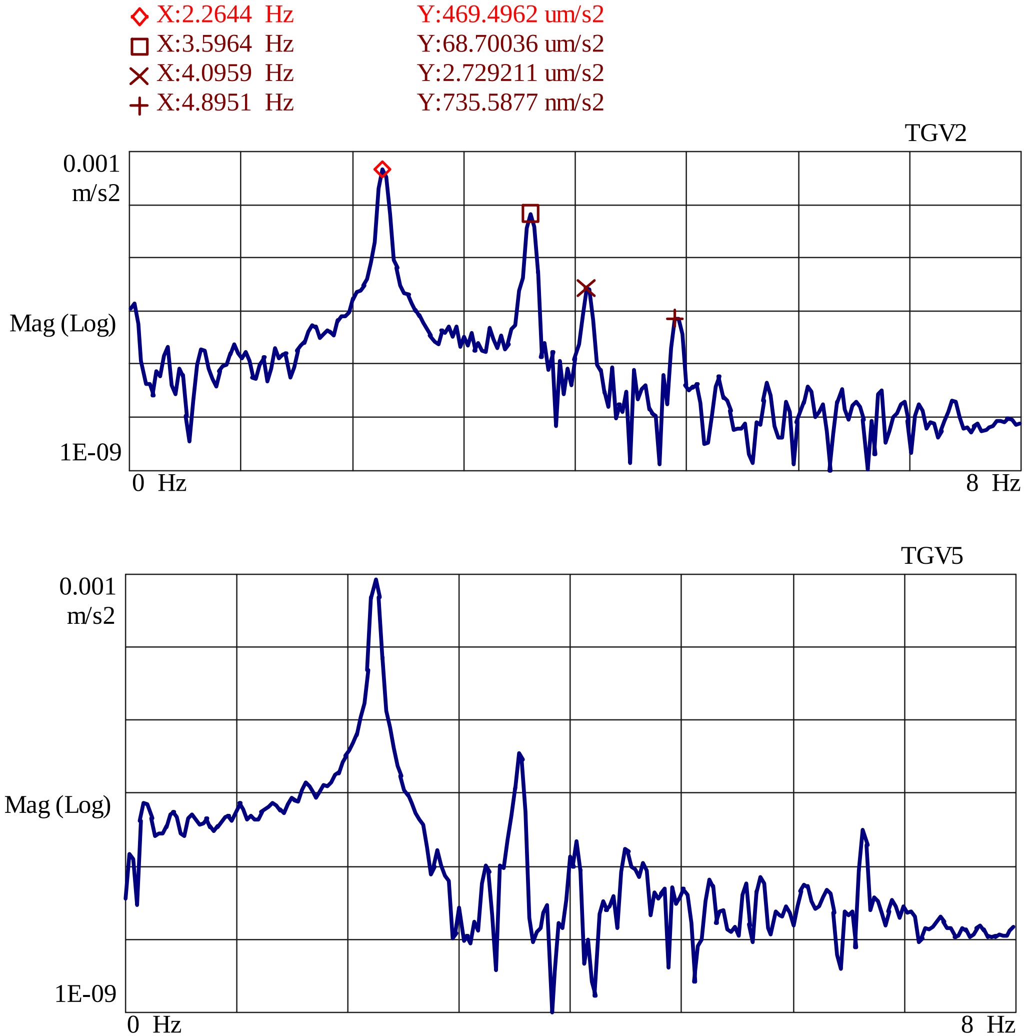
fe=512 Hz

30 times decimated signal

fe=17 Hz

Application to real life signals

- Spectra
  - very noisy
  - 2 leading modes at 2.3 & 3.6 Hz
  - 2 other modes at 4.1 & 4.9 Hz (not always excited)
  - first torsion mode at 3.3 Hz not excited



Application to real life signals

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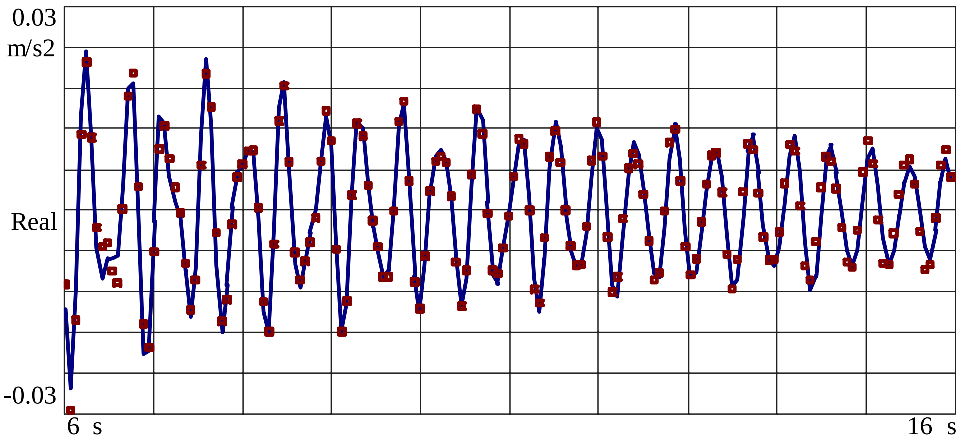
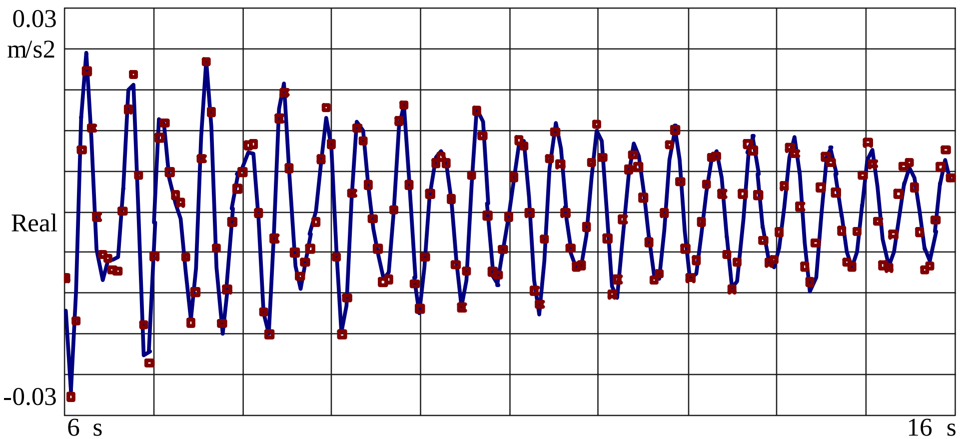
- identification with AR and Eigen model
  - red points give the synthetized curve with model results
  - a fit error is computed and gives model confidence

AR model identification



Eigen model identification



Application to real life signals

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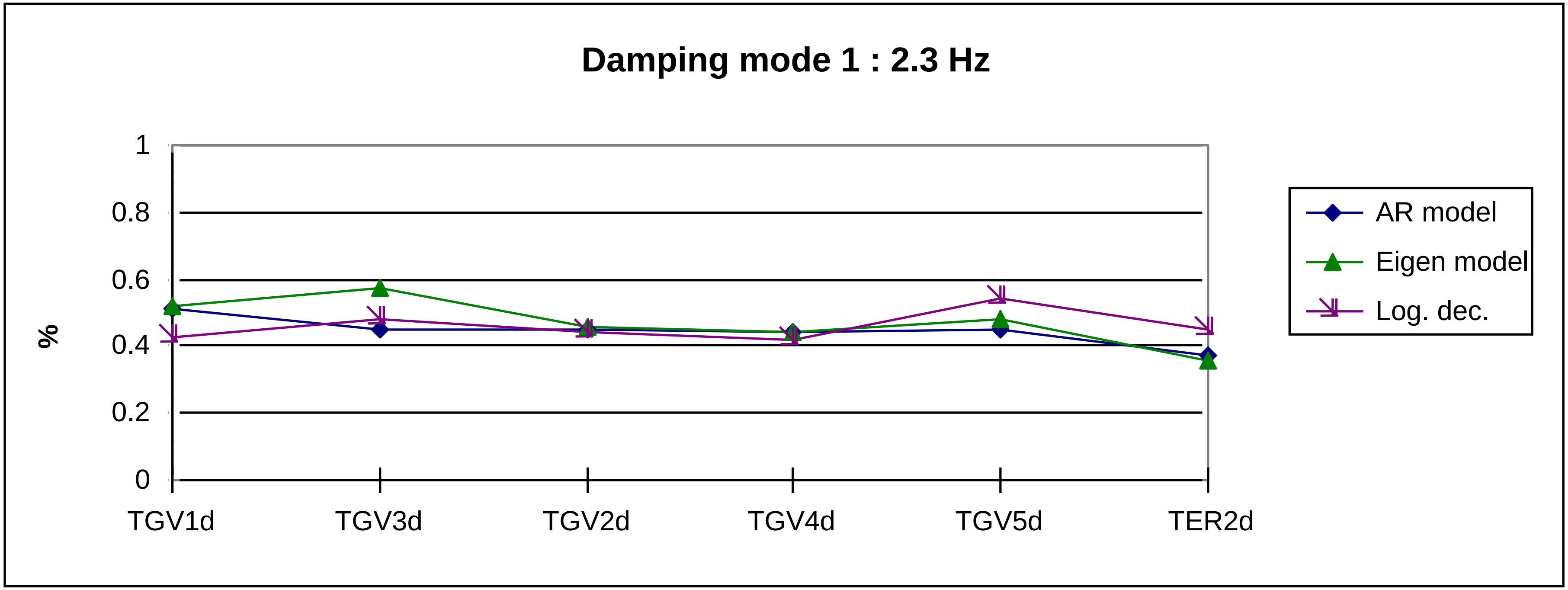
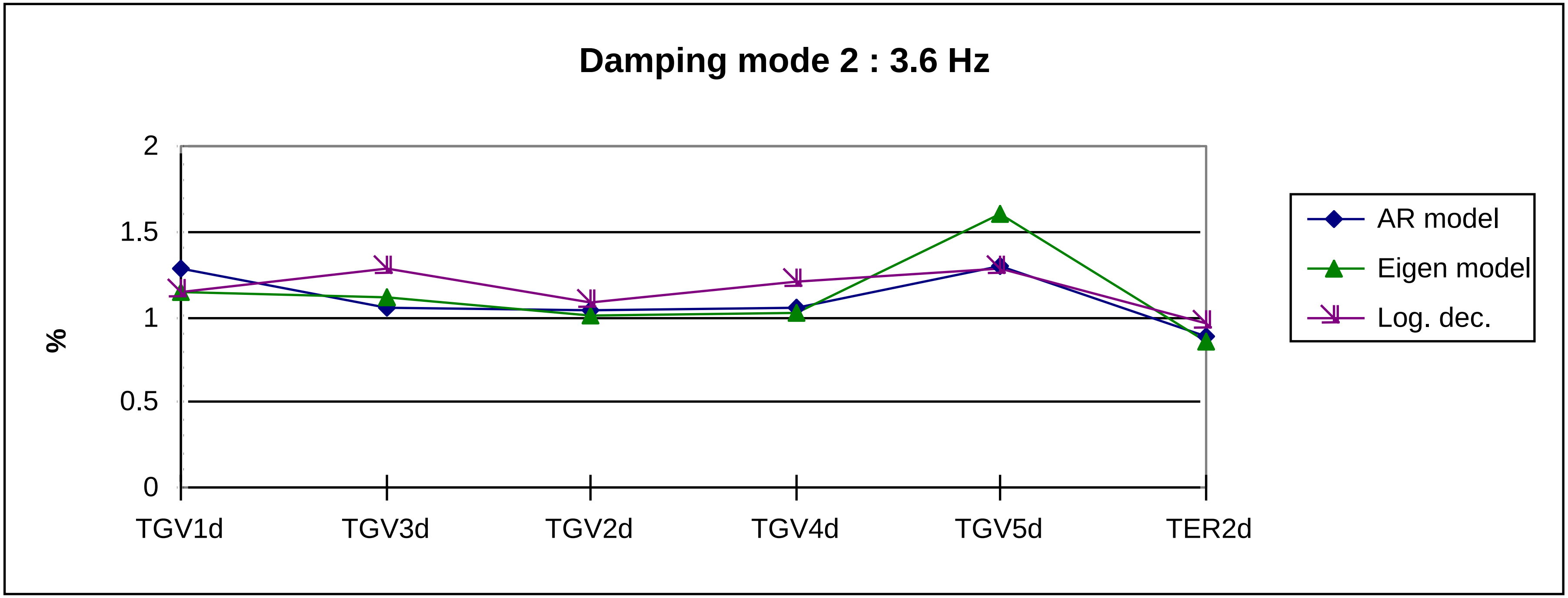
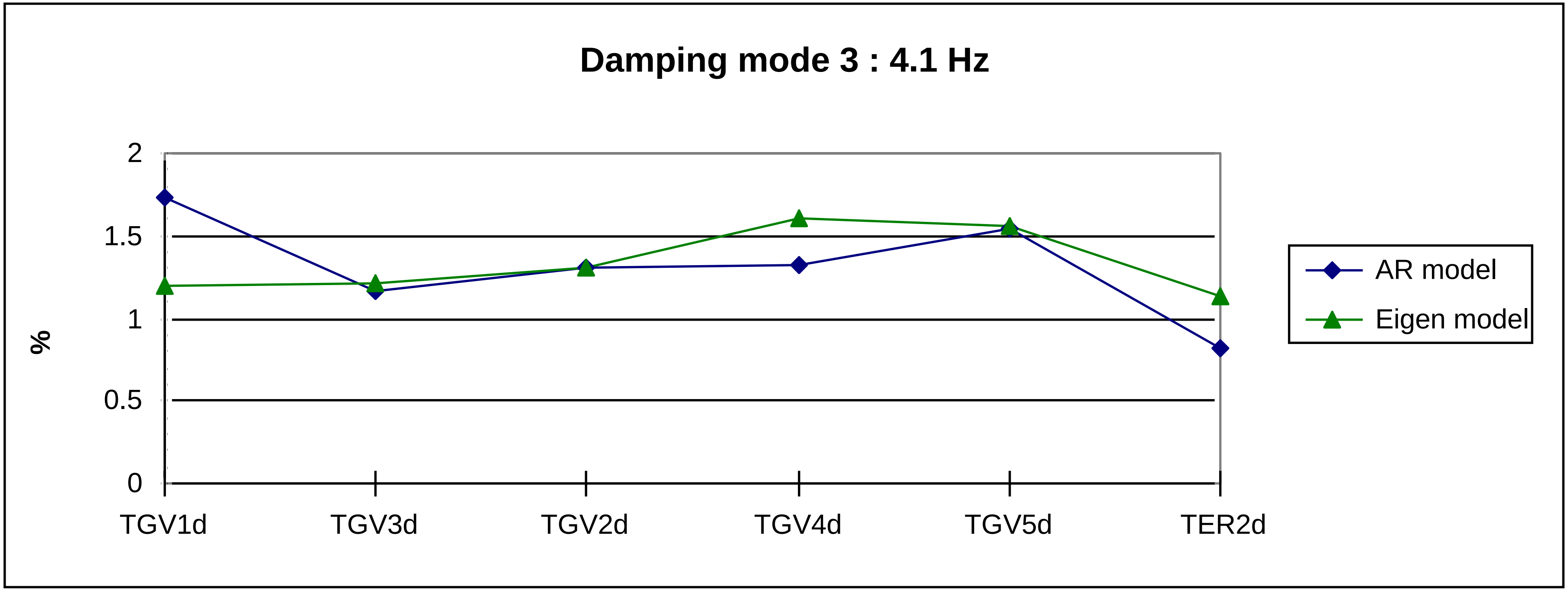


Application to real life signals



Application to real life signals



Application to real life signals

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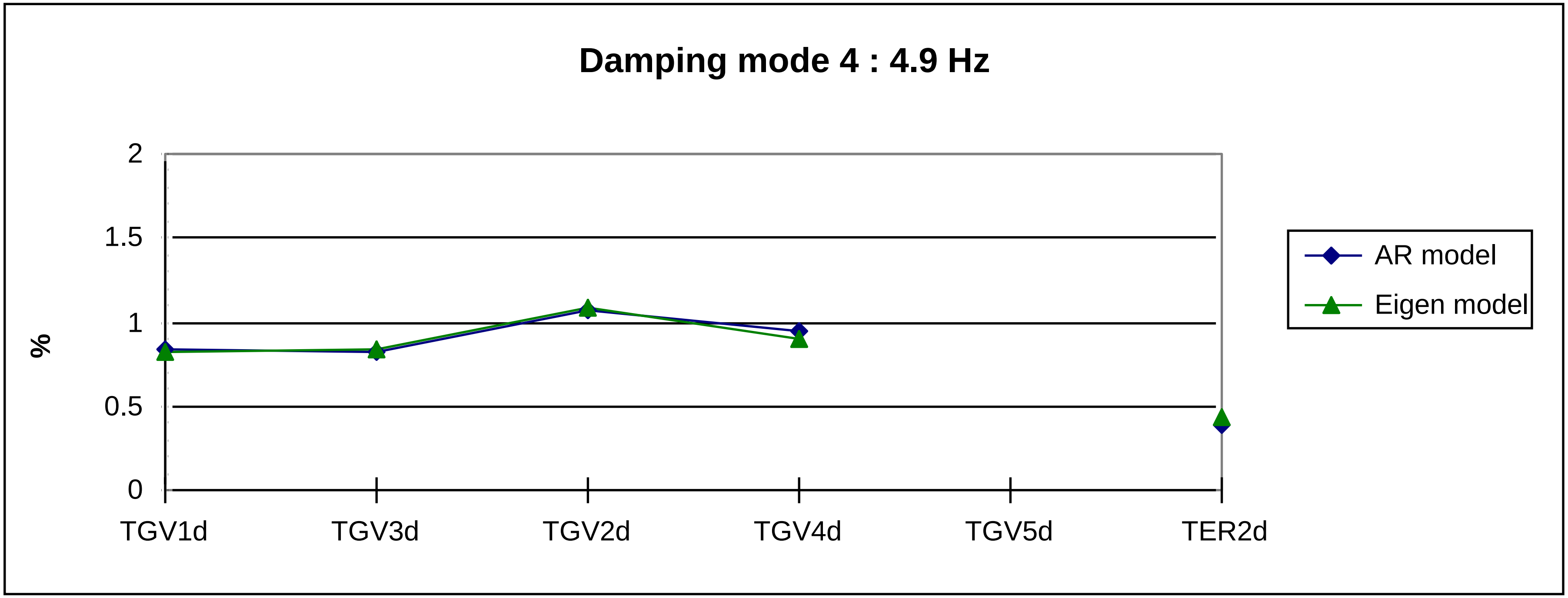

Application to real life signals

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	AR model		**Eigen model**		**logarithmic decrement**	
	damp. (%)	scatter (%)	damp. (%)	scatter (%)	damp. (%)	scatter (%)
mode 1 2.3 Hz	0.45	10	0.47	15	0.46	10
mode 2 3.6 Hz	1.1	15	1.13	22	1.16	11
mode 3 4.1 Hz	1.32	24	1.33	15	not computable	
mode 4 4.9 Hz	0.81	32	0.81	30		

Application to real life signals

Result synthesis

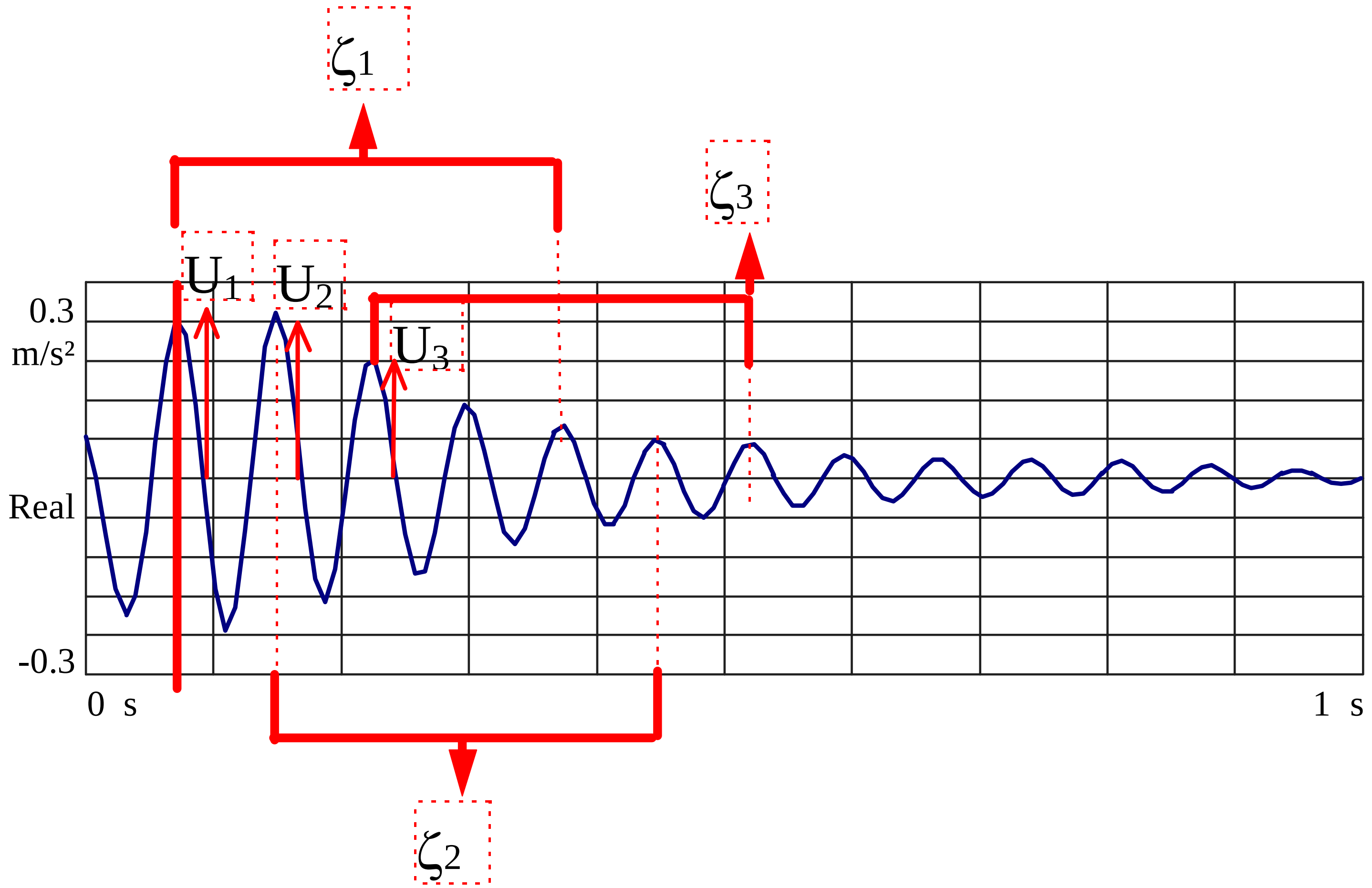
- frequencies are estimated with a very good accuracy ~1 %
- modes 1 & 2 are estimated with maximum 10-20 % dispersion
- modes 3 & 4 are estimated with maximum 30 % dispersion
- discrepancies are caused by measurement/estimation errors AND bridge and excitation characteristics (non linearities, train loading)
- as modes 3 & 4 are only slightly emerging from noise, dispersion is greater

Damping along free decay

- Due to non linearities, damping varies along decay : the change in damping can be estimated using the following procedure
  - the damping is estimated using the Prony method in steps of ~ 10 periods (filtering 48 dB/oct. at 1.5 X first frequency when possible)
  - maximum of amplitude is measured : this amplitude is associated with a damping value

→ Plot of the damping as a function of amplitude

Damping along free decay



Briollay bridge

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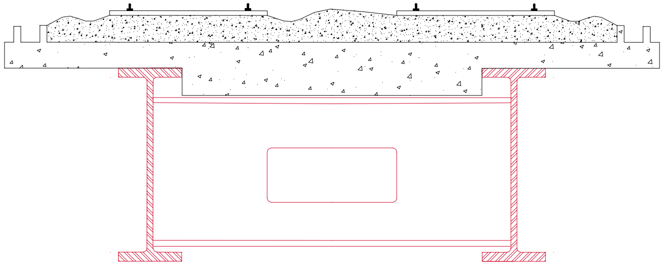
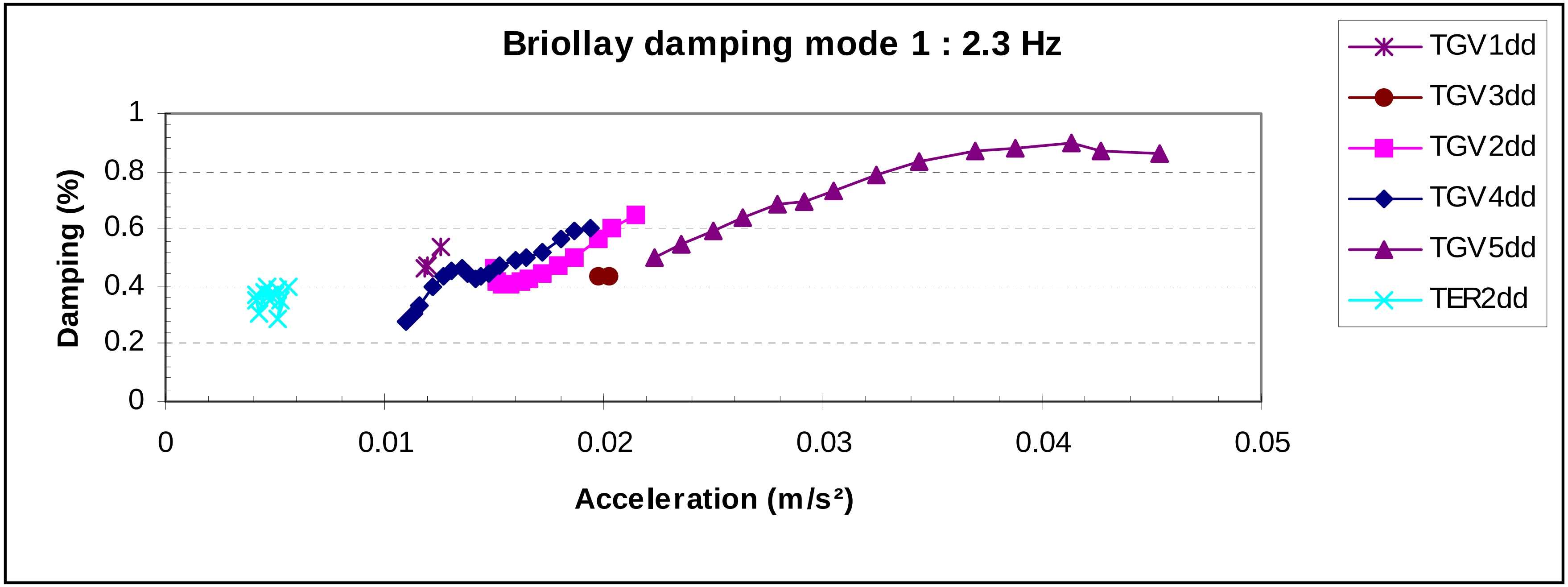
- 3 spans steel/concrete decks
  - strong dependance on amplitude
  - $0.3 < \zeta < 0.9$ %

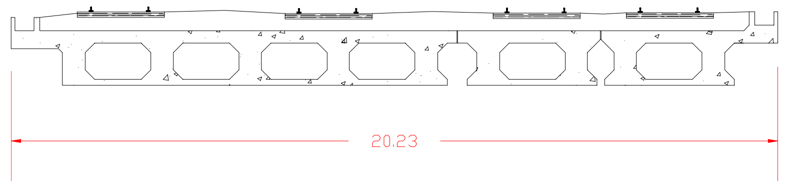


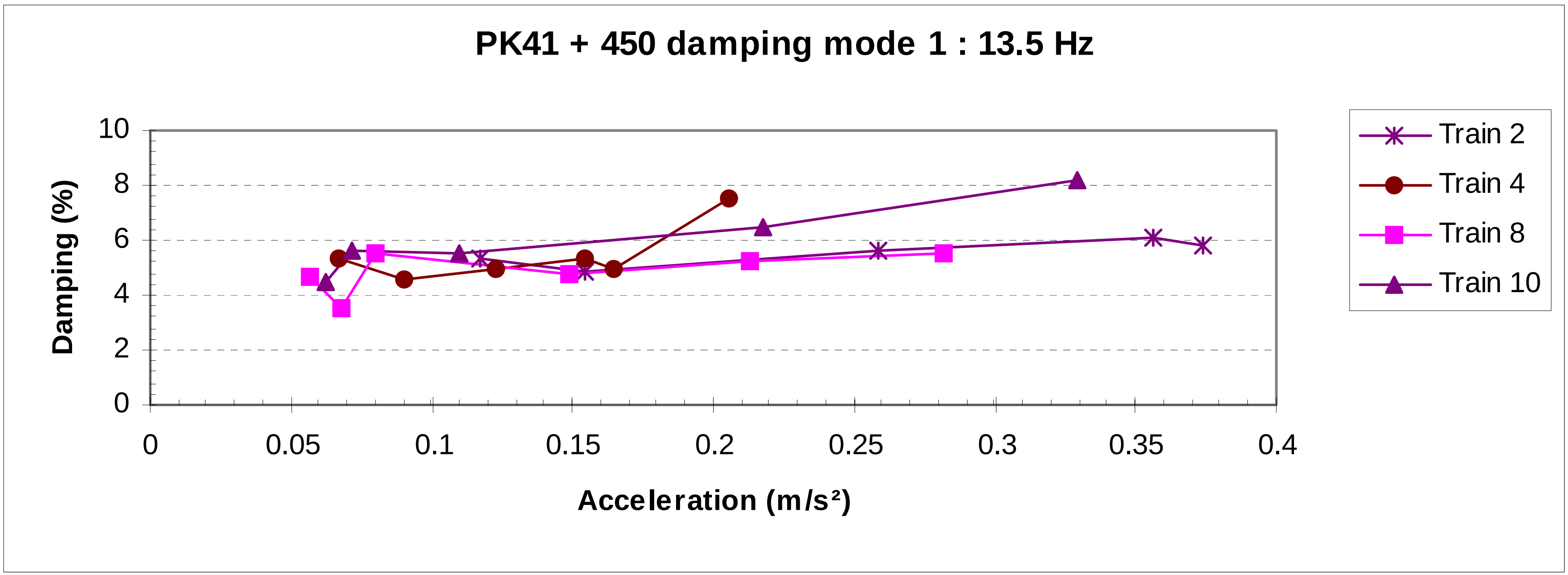


PK 41 + 450

■ 16.5 m long reinforced concrete deck

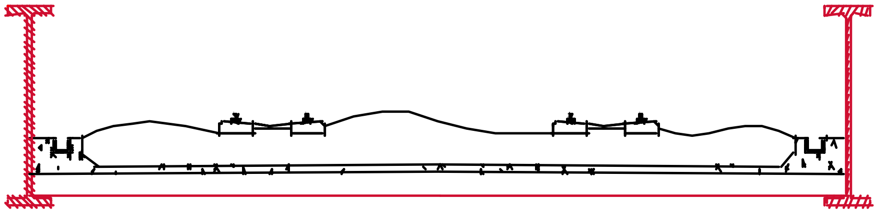
- 4 < ζ < 8 %
- slight increase with acceleration

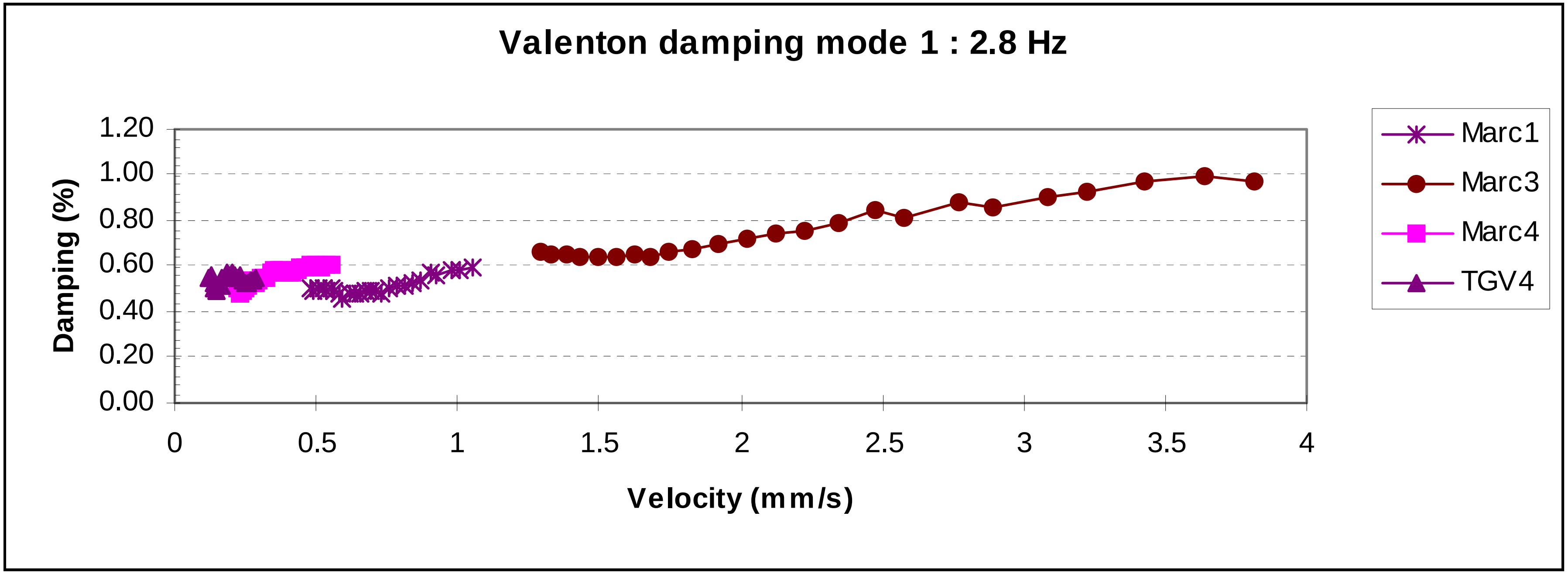




Valenton bridge

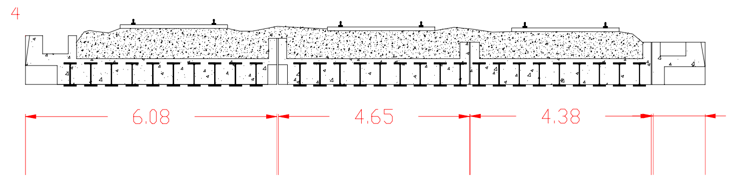
- 46 m long steel deck
  - strong dependance on amplitude
  - good correlation between trains

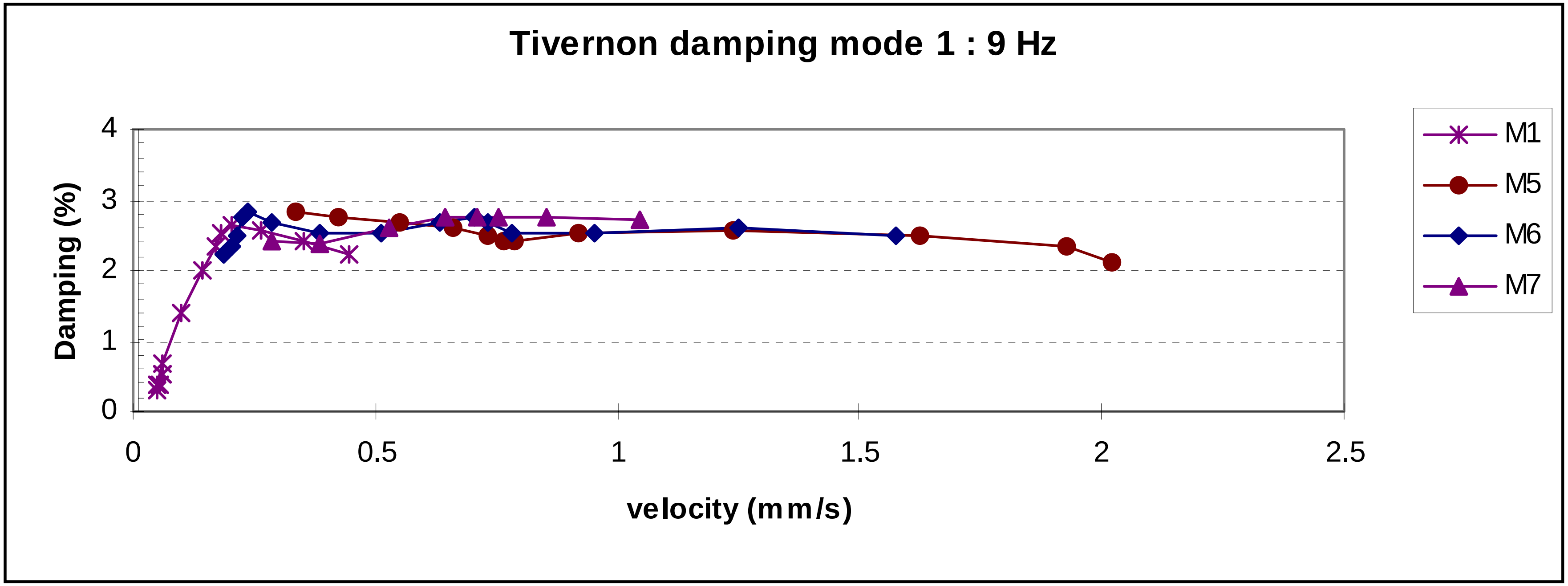




Tivernon bridge

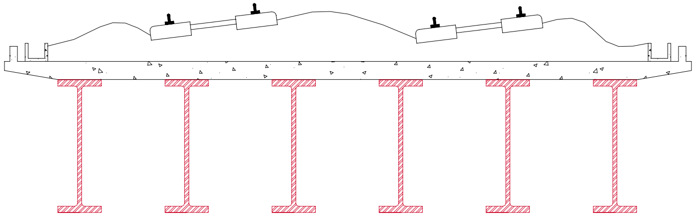
- 3 spans 11.7 m long steel/concrete decks
  - constant damping (except for 1 train)
  - good correlation between trains

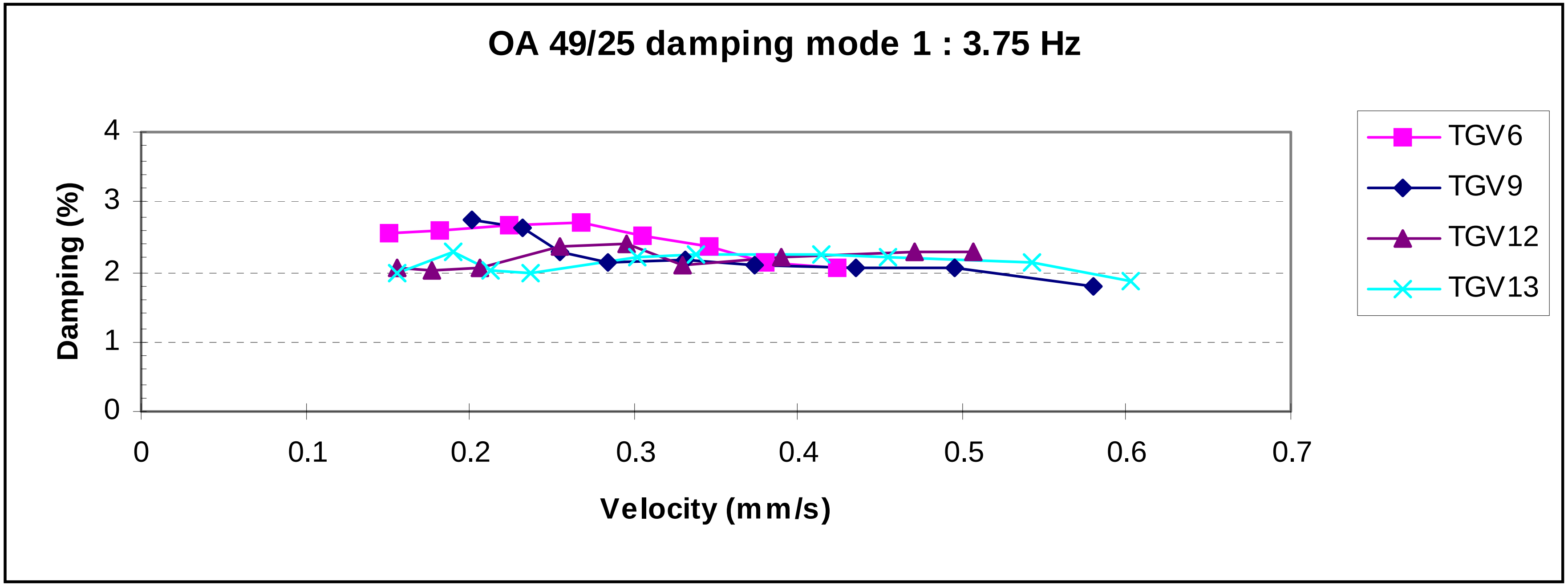




OA 49/25

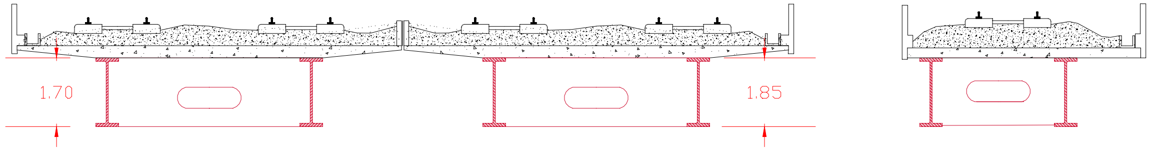
- 46 m long steel/concrete deck
  - constant damping
  - good correlation between trains

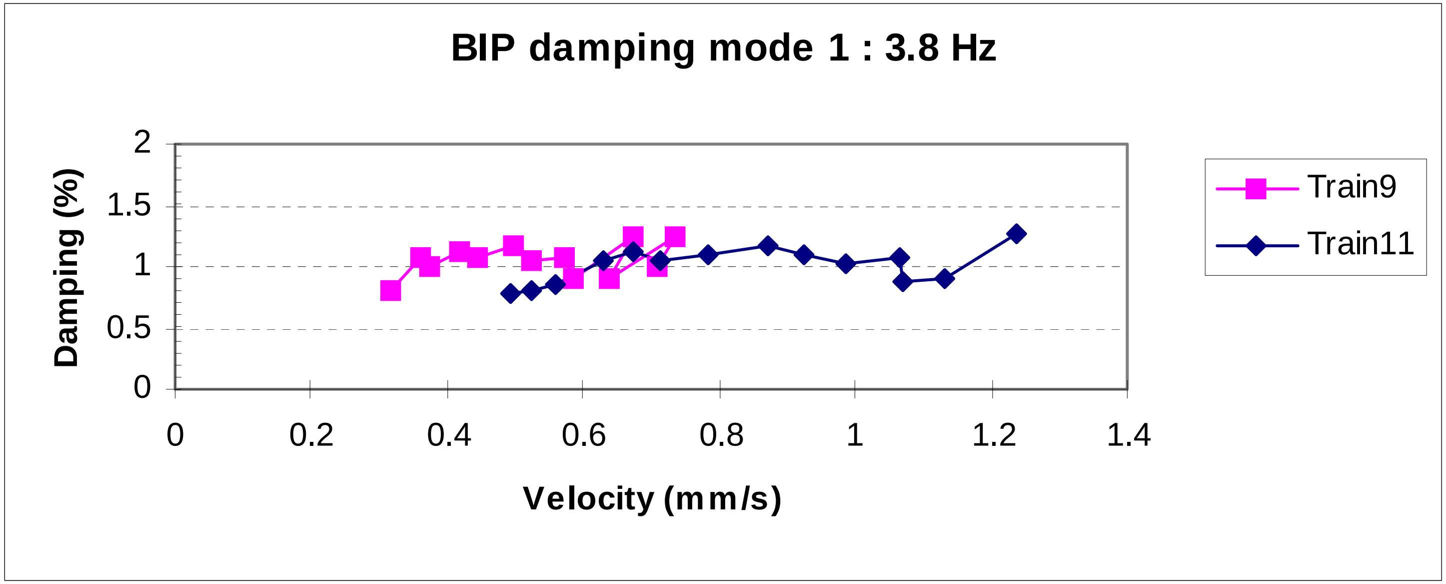




BIP

- 2 spans 33-35 m long steel/concrete decks
  - damping constant





Damping along free decay

■ Conclusion

- two types of behaviour
  - non linearity : damping increases linearly with amplitude
  - constant damping
- due to the deformation amplitude, different train characteristics and speeds lead to different results
  - Briollay : TER 0.25 % ; TGV 0.4 -> 1%
- This kind of analysis gives good confidence on measurements and damping values
  - scattering in measurements can be explained

Conclusion

- Prony and AR models have proved to be very efficient
  - good accuracy in results
  - coupled modes identification
- Recommendations in damping estimation
  - proper signal filtering is of great help
  - damping has to be estimated by a statistical approach
    - use several methods (log. dec. AR eigen method) in order to allow assessment of errors made in the estimation procedure
    - use several measurements to estimate random errors
    - mean and standard deviation must be given

Conclusion

- as damping may vary with vibration level
  - it should be studied throughout the decay period (when possible)
  - damping values must be associated with a deck vibration amplitude
  - trains type and speeds are of great importance in assessing the reproductibility of results